Blackhawk School District

CURRICULUM

Course Title: Pre-Algebra

Grade Level(s): Seventh and Eighth

Length of Course: Year

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COURSE DESCRIPTION:

Pre-Algebra is an introductory course that lays the foundation for the study of Algebra. Students learn about the language of algebra, its properties, and methods of solving equations. Students then build on their foundation of algebra so that it includes rational numbers. Finally, students investigate functions and graphs to model real-world situations and use their algebra skills to solve geometry problems.

Common Core State Standards for Mathematics

Research studies of mathematics education have determined that mathematics curriculum must be more focused and coherent. The Common Core State Standards for Mathematics define what students should understand and be able to do in their study of math. The following Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'Processes and proficiencies" with longstanding importance in mathematics education.

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and

they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

   Mathematically proficient students make sense of quantities and their relationships in problem situations. They bringtwo complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about date, making plausible arguments that take into account the context from which the date arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

   Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or us a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify *relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.*

6. **Attend to precision.**
   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. **Look for and make use of structure.**
   Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Late, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 +7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
8. **Look for and express regularity in repeated reasoning.**
   Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x +1)$, $(x – 1)(x^3+ x^2 + x +1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Special Notes for PSSA purposes: Underlined objectives are for 8th grade students only.
Italicized objectives are for 7th grade students only.

Unit Breakdown	**Objectives**	**PSSA Standards**
Number Theory	**Solve real-world and mathematical problems involving the four operations with rational numbers.** • *Apply properties of operations to add and subtract rational numbers, including real-world contexts.* • *Represent addition and subtraction on a horizontal or vertical number line.* • *Apply properties of operations to multiply and divide rational numbers, including real-world contexts; demonstrate that the decimal form of a rational number terminates or eventually repeats.* **Apply concepts of rational and irrational numbers.** • Determine whether a number is rational or irrational. For rational numbers, show that the decimal expansion terminates or repeats (limit repeating decimals to thousandths). • Convert a terminating or repeating decimal into a rational number (limit repeating decimals to thousandths). • Estimate the value of irrational numbers without a calculator (limit whole number radicand to less than 144). Example: √5 is between 2 and 3 but closer to 2. • Use rational approximations of irrational numbers to compare and order irrational numbers. • Locate/identify rational and irrational numbers at their approximate locations on a number line.	• M07.A-N.1.1.1 • M07.A-N.1.1.2 • M07.A-N.1.1.3 • M08.A-N.1.1.1 • M08.A-N.1.1.2 • M08.A-N.1.1.3 • M08.A-N.1.1.4 • M08.A-N.1.1.5
Expressions and Equations	**Use properties of operations to generate equivalent expressions.** • Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. Example 1: The expression 1/2 • (x + 6) is equivalent to 1/2 • x + 3. Example 2: The expression 5.3 – y + 4.2 is equivalent to 9.5 – y (or –y + 9.5). Example 3: The expression 4w – 10 is equivalent to 2(2w – 5). **Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers.** • *Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. Example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 (or 1.1 × $25 = $27.50).*	• M07.B-E.1.1.1 • M07.B-E.2.1.1 • M07.B-E.2.2.1 • M07.B-E.2.2.2 • M07.B-E.2.3.1

	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems. • *Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* • *Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers, and graph the solution set of the inequality. Example: A salesperson is paid $50 per week plus $3 per sale. This week she wants her pay to be at least $100. Write an inequality for the number of sales the salesperson needs to make, and describe the solutions.* **Determine the reasonableness of the answer(s) in problem-solving situations.** • *Determine the reasonableness of an answer(s), or interpret the solution(s) in the context of the problem. Example: If you want to place a towel bar that is 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **Represent and use expressions and equations to solve problems involving radicals and integer exponents.** • Apply one or more properties of integer exponents to generate equivalent numerical expressions without a calculator (with final answers expressed in exponential form with positive exponents). Properties will be provided. *Example:* $3^{12} \times 3^{-15} = 3^{-3} = \frac{1}{3^3}$ • Use square root and cube root symbols to represent solutions to equations of the form *x*2 = *p* and *x*3 = *p*, where *p* is a positive rational number. Evaluate square roots of perfect squares (up to and including 12^2) and cube roots of perfect cubes (up to and including 53) without a calculator. *Example: If x2 = 25 then x = ±√25.* • Estimate very large or very small quantities by using numbers expressed in the form of a single digit times an integer power of 10, and express how many times larger or smaller one number is than another. *Example: Estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger than the United States population.* • Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Express answers in scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been	• M08.B-E.1.1.1 • M08.B-E.1.1.2 • M08.B-E.1.1.3 • M08.B-E.1.1.4 • M08.B-E.2.1.1 • M08.B-E.2.1.2 • M08.B-E.2.1.3 • M08.B-E.3.1.1 • M08.B-E.3.1.2 • M08.B-E.3.1.3 • M08.B-E.3.1.4 • M08.B-E.3.1.5

	generated by technology (e.g., interpret 4.7EE9 displayed on a calculator as 4.7 × 109). **Analyze and describe linear relationships between two variables, using slope.** • Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. • Use similar right triangles to show and explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. • Derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. **Write, solve, graph, and interpret linear equations in one or two variables, using various methods.** • Write and identify linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). • Solve linear equations that have rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. • Interpret solutions to a system of two linear equations in two variables as points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. • Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. Example: 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. • Solve real-world and mathematical problems leading to two linear equations in two variables. Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	
Functions	**Define, evaluate, and compare functions displayed algebraically, graphically, numerically in tables, or by verbal descriptions.** • Determine whether a relation is a function. • Compare properties of two functions each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions). Example: Given a linear function represented by a table of values and a linear	• M08.B-F.1.1.1 • M08.B-F.1.1.2 • M08.B-F.1.1.3

	function represented by an algebraic expression, determine which function has the greater rate of change. • Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear. **Represent or interpret functional relationships between quantities using tables, graphs, and descriptions.** • Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch or determine a graph that exhibits the qualitative features of a function that has been described verbally.	• M08.B-F.2.1.1 • M08.B-F.2.1.2
Ratios and Proportional Relationships	**Analyze, recognize, and represent proportional relationships and use them to solve real-world and mathematical problems.** • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. Example: If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour. • Determine whether two quantities are proportionally related (e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). • Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. • Represent proportional relationships by equations. Example: If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. • Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. • Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease.	• M07.A-R.1.1.1 • M07.A-R.1.1.2 • M07.A-R.1.1.3 • M07.A-R.1.1.4 • M07.A-R.1.1.5 • M07.A-R.1.1.6

Geometry	**Describe and apply properties of geometric figures.** • *Solve problems involving scale drawings of geometric figures, including finding length and area.* • *Identify or describe the properties of all types of triangles based on angle and side measure.* • *Use and apply the triangle inequality theorem.* • *Describe the two-dimensional figures that result from slicing three-dimensional figures. Example: Describe plane sections of right rectangular prisms and right rectangular pyramids.* **Identify, use and describe properties of angles and their measures.** • *Identify and use properties of supplementary, complementary and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.* • *Identify and use properties of angles formed when two parallel lines are cut by a transversal (e.g., angles may include alternate interior, alternate exterior, vertical, corresponding).* **Determine circumference, area, surface area, and volume.** • *Find the area and circumference of a circle. Solve problems involving area and circumference of a circle(s). Formulas will be provided.* • *Solve real-world and mathematical problems involving area, volume, and surface area of two and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Formulas will be provided.* **Apply properties of geometric transformations to verify congruence or similarity.** • Identify and apply properties of rotations, reflections, and translations. Example: Angle measures are preserved in rotations, reflections, and translations. • Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them. • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures, using coordinates. • Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. **Solve problems involving right triangles by applying the Pythagorean theorem.** • Apply the converse of the Pythagorean Theorem to show a triangle is a right triangle. • Apply the Pythagorean theorem to determine unknown side lengths in right	• M07.C-G.1.1.1 • M07.C-G.1.1.2 • M07.C-G.1.1.3 • M07.C-G.1.1.4 • M07.C-G.2.1.1 • M07.C-G.2.1.2 • M07.C-G.2.2.1 • M07.C-G.2.2.2 • M07.C-G.3.1.1 • M08.C-G.1.1.1 • M08.C-G.1.1.2 • M08.C-G.1.1.3 • M08.C-G.1.1.4 • M08.C-G.2.1.1 • M08.C-G.2.1.2 • M08.C-G.2.1.3 • M08.C-G.3.1.1

	triangles in real-world and mathematical problems in two and three dimensions. (Figures provided for problems in three dimensions will be consistent with Eligible Content in grade 8 and below.) • Apply the Pythagorean theorem to find the distance between two points in a coordinate system. **Apply volume formulas of cones, cylinders, and spheres.** • Apply formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems. Formulas will be provided.	
Statistics and Probability	**Use random samples.** • *Determine whether a sample is a random sample given a real-world situation.* • *Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.* *Example 1: Estimate the mean word length in a book by randomly sampling words from the book.* *Example 2: Predict the winner of a school election based on randomly sampled survey data.* **Use statistical measures to compare two numerical data distributions.** • *Compare two numerical data distributions using measures of center and variability.* *Example 1: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team. This difference is equal to approximately twice the variability (mean absolute deviation) on either team. On a line plot, note the difference between the two distributions of heights.* *Example 2: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth grade science book.* **Predict or determine the likelihood of outcomes.** • *Predict or determine whether some outcomes are certain, more likely, less likely, equally likely, or impossible (i.e., a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event).* **Use probability to predict outcomes.** • *Determine the probability of a chance event given relative frequency. Predict the approximate relative frequency given the probability.* *Example: When rolling a number cube 600 times, predict that a 3 or 6 would*	• M07.D-S.1.1.1 • M07.D-S.1.1.2 • M07.D-S.2.1.1 • M07.D-S.3.1.1 • M07.D-S.3.2.1 • M07.D-S.3.2.2 • M07.D-S.3.2.3

	be rolled roughly 200 times, but probably not exactly 200 times. • *Find the probability of a simple event, including the probability of a simple event not occurring. Example: What is the probability of not rolling a 1 on a number cube?* • *Find probabilities of independent compound events using organized lists, tables, tree diagrams, and simulation.* **Analyze and interpret bivariate data displayed in multiple representations.** • <u>Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative correlation, linear association, and nonlinear association.</u> • <u>For scatter plots that suggest a linear association, identify a line of best fit by judging the closeness of the data points to the line.</u> • <u>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</u> **Understand that patterns of association can be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.** • <u>Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible associations between the two variables. Example: Given data on whether students have a curfew on school nights and whether they have assigned chores at home, is there evidence that those who have a curfew also tend to have chores?</u>	• M08.D-S.1.1.1 • M08.D-S.1.1.2 • M08.D-S.1.1.3 • M08.D-S.1.2.1